G. Matter Waves

- Electron (matter! not light! Discovered as a particle by J.J. Thomson 1897)
- Take electron as our model matter
  [quantum problems: e^- in atoms, molecules, solids]
- Key experimental Fact
  - Electrons show exactly the same behavior as light (photons) in experiments (interference) under appropriate situations
    [Wave nature of particle]
  - Implications on physics as discussed for light (photons) and the role of wave theory are also valid for electrons

- Physics is an experimental science – Let's see some exp'tal results

Electrons in **single-slit** experiments [Mullenstedt and Jonsson, Z. Physik 155 (1959) 472]

Experimental results (1959)

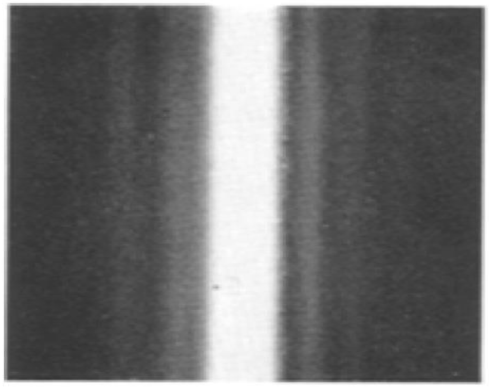
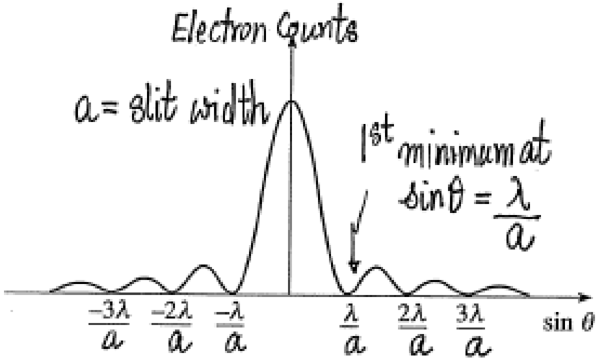
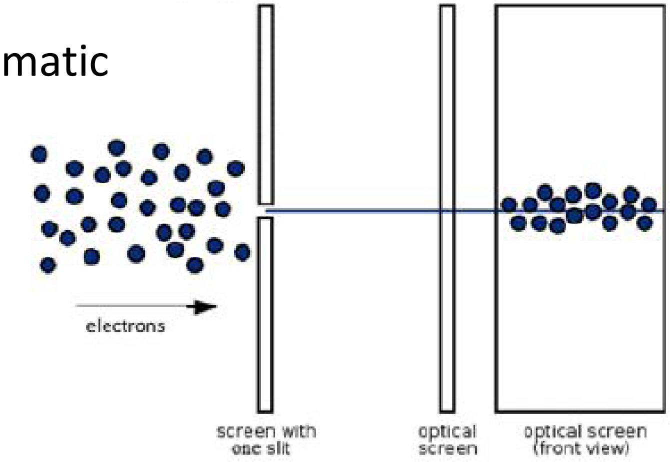


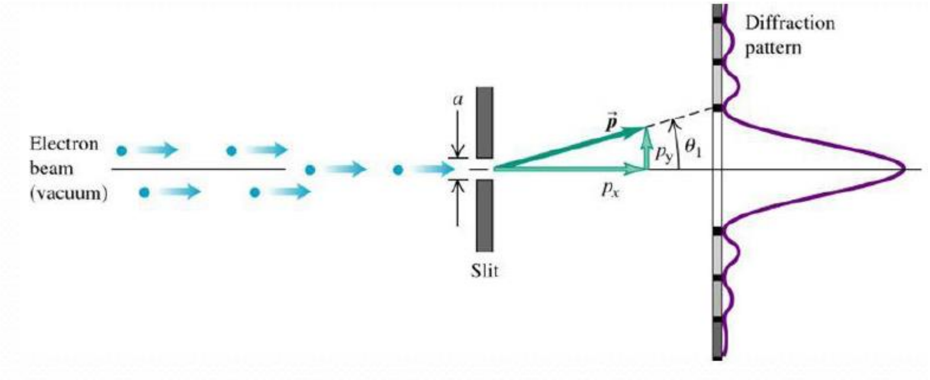
Fig. 7. Elektronenbeugungsaufnahme an einem Spalt (Fraunhofer-Ebene)



Intensity pattern generated by passing through **many electrons** is exactly what is expected of ***waves***

Schematic





- <u>Note</u>: Single-slit experiments interpreted as <u>Wave property</u>.

  First minimum to appear at $\sin\theta = \dfrac{\lambda}{a}$ ← wavelength of "wave" in exp't

  ← width of slit

- Electrons' kinetic energy T can be tuned by accelerating voltage [gain eV energy]

  $$T = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

  [p = momentum of electrons]

  ← mass of electron

- Repeat experiment for $\underbrace{\text{different k.e.}}_{\text{different } p}$ and $\underbrace{\text{observe first minimum}}_{\text{find } \lambda}$

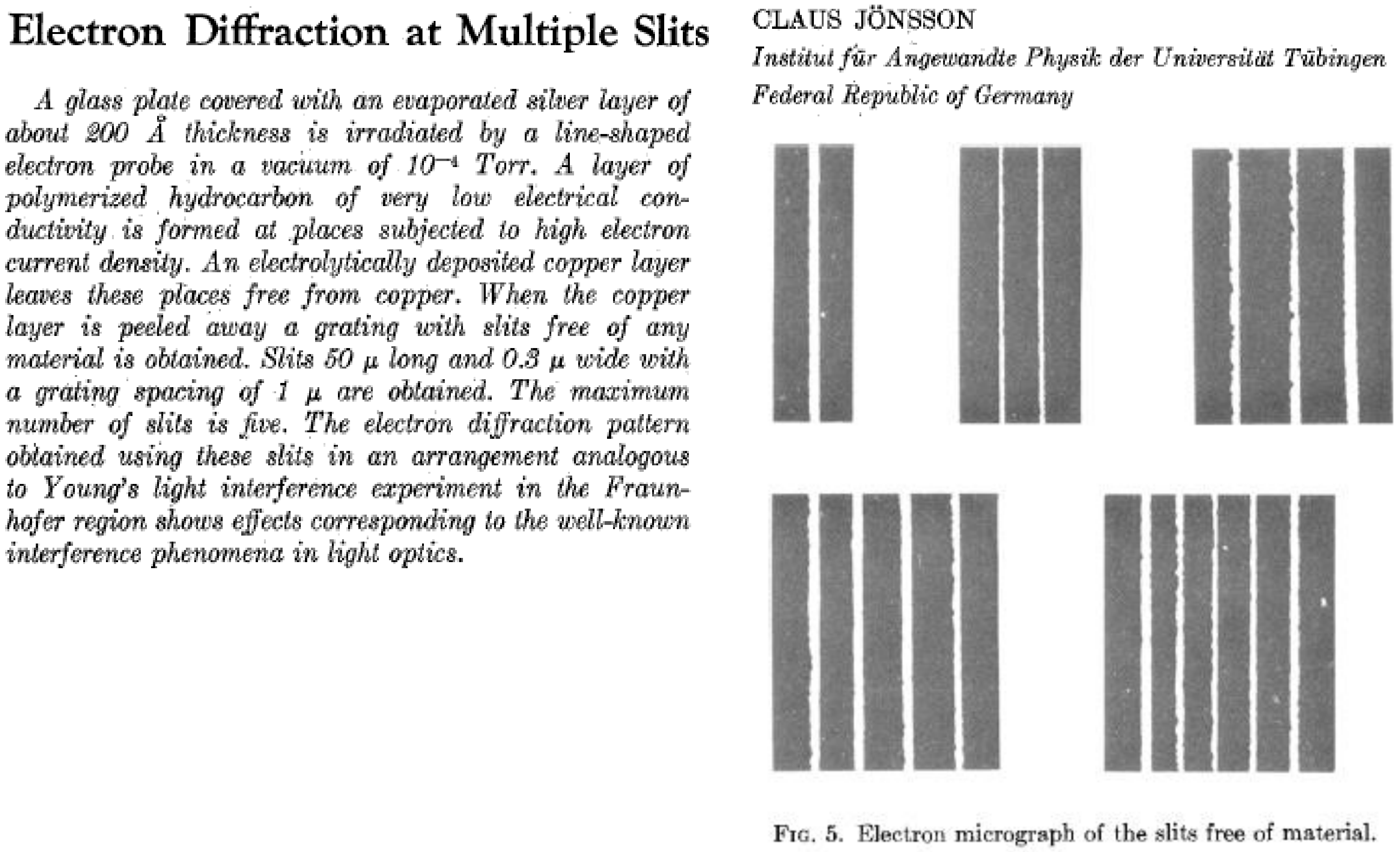
  ⇒ relationship between p and λ

  Result: $$\boxed{\lambda = \frac{h}{p} = \frac{2\pi\hbar}{p}}$$

  - Proposed by de Broglie (~1923)
  - $\lambda = \lambda_{dB}$ = de Broglie Wavelength
  - Verified by exp'ts over ~100 years!

Electrons in **two-slit and multiple slits** experiments [Jonsson, Z. Physik 161 (1961) 452]

[English translation: American Journal of Physics 42 (1974) 4]



Electron Diffraction at Multiple Slits

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A glass plate covered with an evaporated silver layer of about 200 Å thickness is irradiated by a line-shaped electron probe in a vacuum of 10^{-4} Torr. A layer of polymerized hydrocarbon of very low electrical conductivity is formed at places subjected to high electron current density. An electrolytically deposited copper layer leaves these places free from copper. When the copper layer is peeled away a grating with slits free of any material is obtained. Slits 50 μ long and 0.3 μ wide with a grating spacing of 1 μ are obtained. The maximum number of slits is five. The electron diffraction pattern obtained using these slits in an arrangement analogous to Young's light interference experiment in the Fraunhofer region shows effects corresponding to the well-known interference phenomena in light optics.

FIG. 5. Electron micrograph of the slits free of material.

Link to Jonsson's paper: http://aapt.scitation.org/doi/abs/10.1119/1.1987592

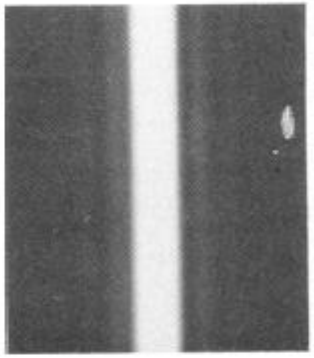


FIG. 7. Electron-diffraction photograph from a single slit

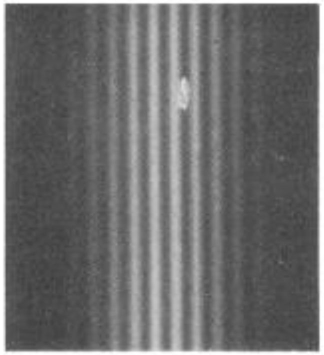


FIG. 8. Electron-diffraction photograph from two slits

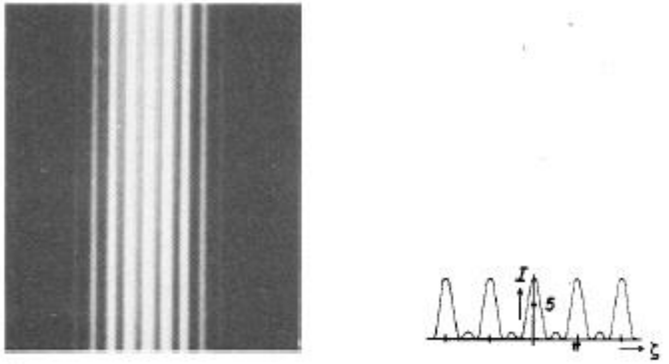


FIG. 9. Electron-diffraction photograph from three slits, with theoretical intensity curve.

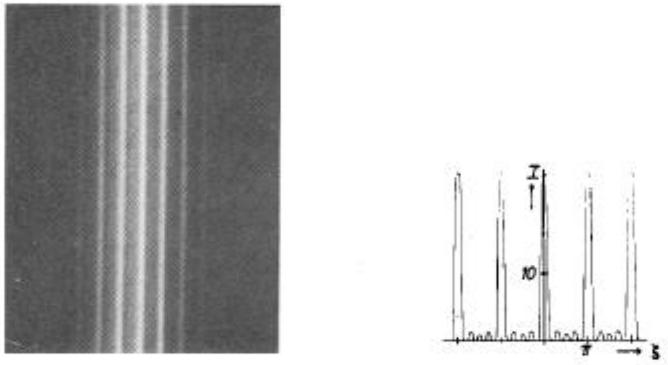


FIG. 10. Electron-diffraction photograph from four slits, with theoretical intensity curve.

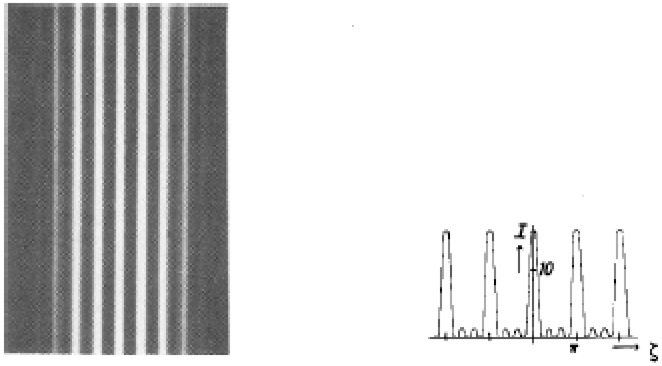
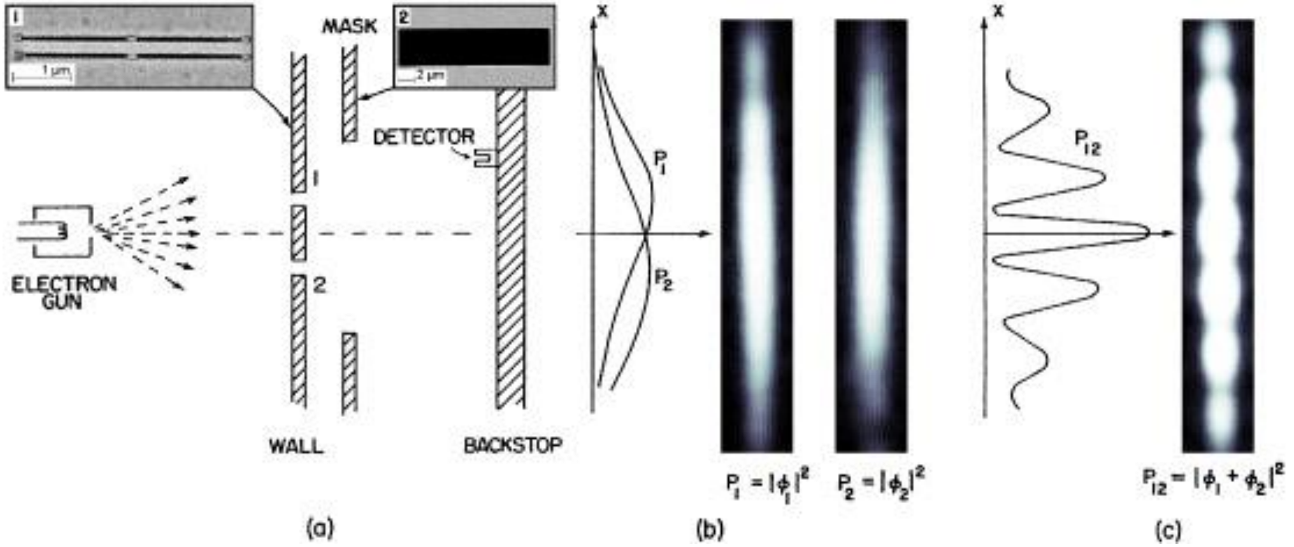


FIG. 11. Electron-diffraction photograph from five slits, with theoretical intensity curve.

- Electrons exhibit wave properties and $\boxed{\lambda = \frac{h}{p} = \frac{2\pi\hbar}{p}}$

More on two-slit experiments with electrons



Picture taken from IOPScience, Institute of Physics, UK

(b) When only slit 1 is open, observed P_1. When only slit 2 is open, observed P_2.

(c) When two slits are open, observed P_{12}.

$$P_{12} \neq P_1 + P_2$$

Analogous to light (interference in action and something is waving)

When you know which slit the electron goes through (e.g. by closing one slit), the interference pattern P_{12} is destroyed.

Need to use waves to describe an electron and a wave theory

Two-slit Experiments

separation between slits

- Interference patterns
- Maxima at $d \sin\theta = n\lambda$ wavelength

[where maxima show up determine λ]

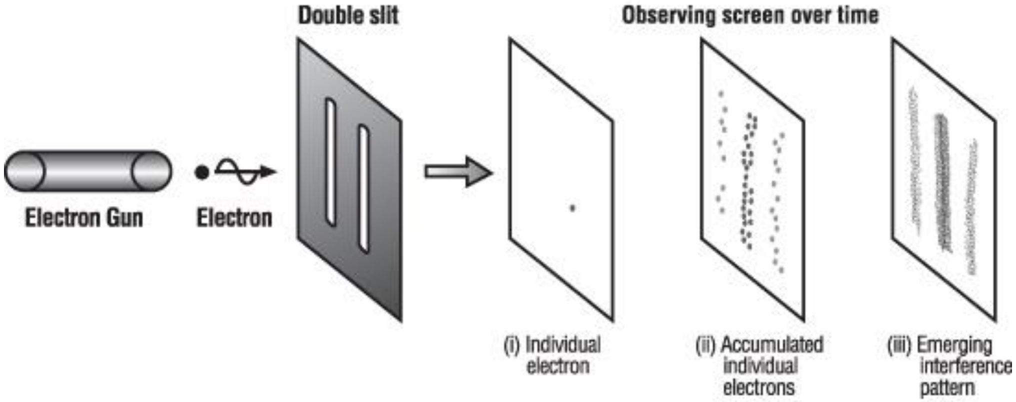
- Vary k.e. [thus $\frac{p^2}{2m}$ ($\therefore$ vary p] and inspect how maxima shift

$\Rightarrow$ relationship between λ and p

Result: $\lambda = \frac{h}{p} = \frac{2\pi\hbar}{p}$ de Broglie wavelength

Two-slit experiments with electrons (dim source) [one electron at a time]

Schematic: Sending one electron into the apparatus at a time until it is detected on the screen

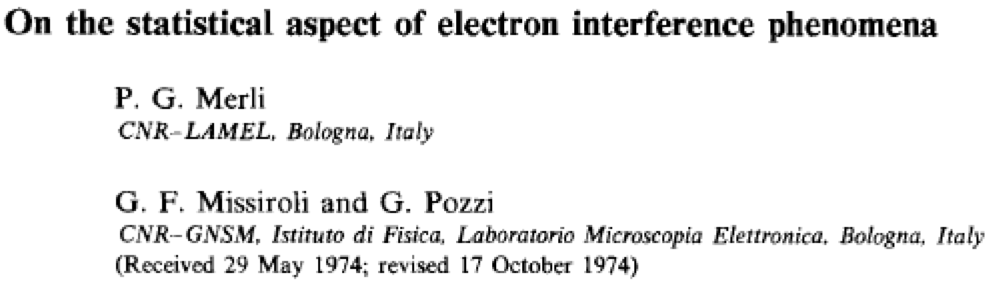


Double-slit apparatus showing the pattern of electron hits on the observing screen building up over time.

- Electron is detected as a particle
- Can’t predict where one electron lands
- Interference pattern shows up after repeating the one-electron-in-apparatus experiment for many times

[Analogous to photons]

Are these real?



On the statistical aspect of electron interference phenomena

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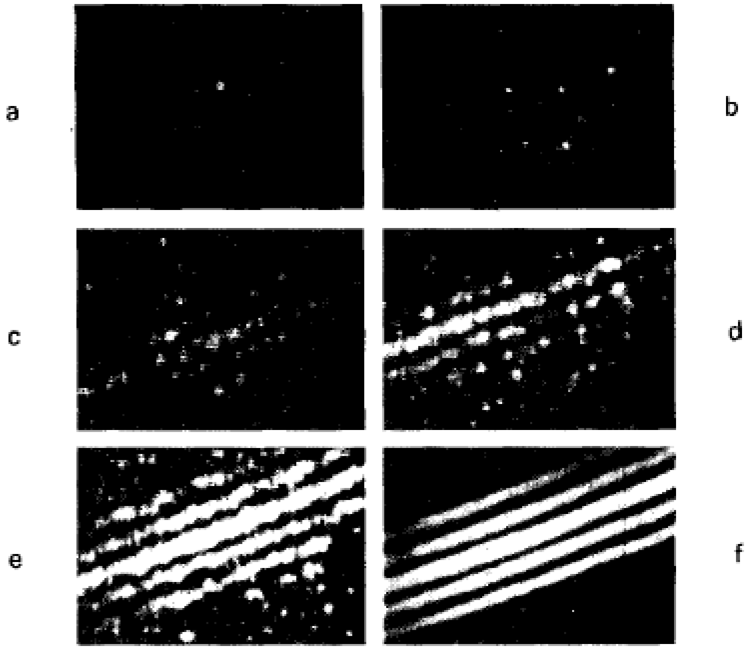
(Received 29 May 1974; revised 17 October 1974)

Am. J. Phys. 44 (1976) 306



- Electron is detected as a particle
- Can’t predict where one electron lands
- Interference pattern shows up after repeating the one-electron-in-apparatus experiment for many times

It is real stuff!

(a) To (f): more electrons hit screen

Link to paper: http://aapt.scitation.org/doi/abs/10.1119/1.10184

Demonstration of single-electron buildup of an interference pattern

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(Received 17 December 1987; accepted for publication 22 March 1988)

[Use a design of transmission electron microscope to do the job]

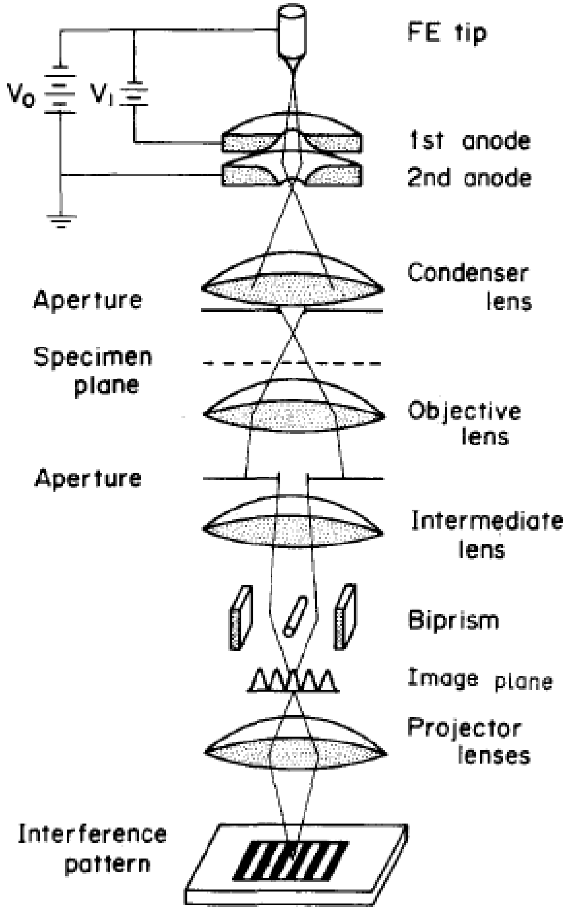


Fig. 3. Electron-optical diagram of the interference experiment.

Am. J. Phys. 57 (1989) 117

Link: http://aapt.scitation.org/doi/abs/10.1119/1.16104

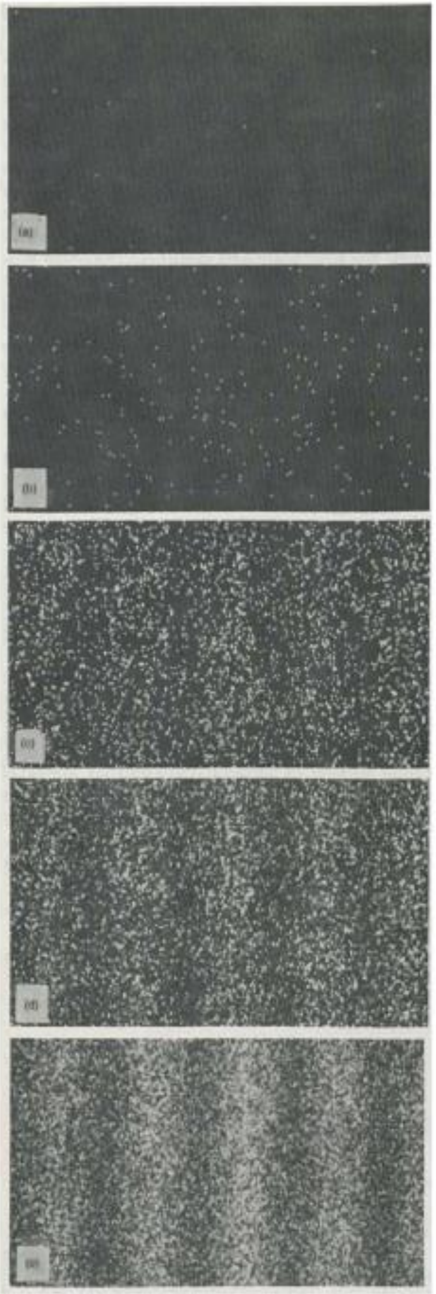


Fig. 5. Buildup of the electron interference pattern. The central field of view, $\frac{1}{2}$ width and $\frac{1}{3}$ length, of the whole field of the detector plane is shown here. The picture extends similarly to the whole field: (a) Number of electrons = 10; (b) Number of electrons = 100; (c) Number of electrons = 3000; (d) Number of electrons = 20 000; and (e) Number of electrons = 70 000.

10 electrons

100 electrons

3000 electrons (started to see some pattern)

20000 electrons

70000 electrons

Electron is detected at a location on the screen one at a time

Cannot predict where an electron lands

Two open slits required for seeing interference pattern (even one electron in apparatus at a time)

Electrons *are* ***no ordinary particles*** (they show wave nature)

Implications for Quantum Theory

- To describe electron[+](s) at atomic (microscopic) scale, we need to invoke a <u>wave description</u>.
- Experiments verified <u>de Broglie</u> (1923) proposal of <u>wave nature of particle</u>

Wave property →
$$\lambda = \frac{h}{p} = \frac{2\pi\hbar}{p}$$
← particle property

(h = Planck's constant $\approx 1.05 \times 10^{-34}$ J·s)

[It says: a particle of <u>definite momentum</u> p ("definite" means certain of having that value) has a definite wavelength λ]

[de Broglie: 1929 Nobel Prize]

[+] An important point to note here is the electron (representative of matter) is <u>massive</u> (has mass), while photon is massless. (thus $E = cp$ for photons).

- Need wave description for matter (particle such as electron)
  - $\Psi(x,t)$ OR $\Psi(x,y,z,t)$ OR $\Psi(\vec{r},t)$

    $\boxed{\text{Wavefunctions}}$ (analogous to $\vec{E}(\vec{r},t)$ in EM theory)
  - What is Waving?

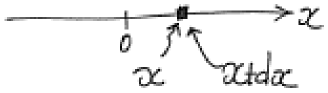
    $\Psi(x,t)$ is waving
  - What is it?

    $|\Psi(x,t)|^2 \sim$ Likelihood of finding particle at location x at time t.

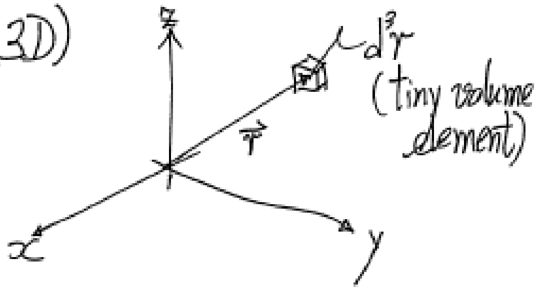
    (analogous to Intensity $\propto |\vec{E}(\vec{r},t)|^2$ in EM)

More precisely,

$|\Psi(x,t)|^2 dx \propto$ Probability of finding the particle in the range x to $x+dx$ at time t (1D)



$|\Psi(\vec{r},t)|^2 \underbrace{d^3r}_{\text{Volume element}} \propto$ Probability of finding the particle to be in a volume element d^3r located at $\vec{r}$ at time t (3D)



Key Point:

Physical Meaning of $\Psi(\vec{r},t)$ is attached to $|\Psi(\vec{r},t)|^2$ but not $\Psi(\vec{r},t)$

- Names

$$\Psi(\vec{r},t) \text{ or } \Psi(x,t) \quad \begin{cases} \text{"Wavefunction"} \\ \text{"probability amplitude"} \ [\because \text{Prob.} \propto |\Psi|^2] \end{cases}$$

Probabilistic role of Wave Theory

Question: What are the units of $\Psi(x,t)$ and $\Psi(\vec{r},t)$?

- Understanding Observations

$$P_{12} \neq P_1 + P_2$$

Slits 1 and 2 are open [interference pattern]

Only slit 1 is open

Only slit 2 is open

- Only slit 1 is open Ψ_1 and $P_1 = |\Psi_1|^2$
- Only slit 2 is open Ψ_2 and $P_2 = |\Psi_2|^2$

<u>Slits 1 and 2 are open</u>

described by $\Psi_{12} = \Psi_1 + \Psi_2$ [analogous to Young's experiment]

$$P_{12} = |\Psi_{12}|^2 = |\Psi_1 + \Psi_2|^2$$

("add amplitudes then square")

Physical meaning is attached to $|\Psi|^2$

$$= \underbrace{|\Psi_1|^2} + \underbrace{|\Psi_2|^2} + \Psi_1^* \Psi_2 + \Psi_2^* \Psi_1$$

$$= P_1 + P_2 + 2\,\mathrm{Re}[\Psi_1^* \Psi_2] \neq P_1 + P_2$$

(interpretation Works!)

Quantum States are unusual (in classical thinking)

Consider $\boxed{\Psi_{12} \propto \Psi_1 + \Psi_2}$ [Recall: $\underset{\text{observed intensity}}{P_{12}} = |\Psi_{12}|^2$]

What does it mean?

- This is the state[†] of the system [electron *to be* detected on screen] *before* the measurement of electron's location is made
- *Before measurement*, that's all we can say $\Psi_{12} \propto (\Psi_1 + \Psi_2)$
  - ⇒ it does *not* have a location waiting to be measured
  - [*Think*: contrast this with the situation that we do have a location [doesn't matter someone find out where you are or not], which is revealed by measurement]

[†] In QM, the wavefunction gives all the information about the *state* of a system. More about states will be discussed later.

- $\Psi_{12} \propto (\Psi_1 + \Psi_2)$

  the system is described by a state that "<u>the particle</u> (if we want to say something about the electron) <u>is in "state 1"</u> [meaning: through slit 1] <u>AND</u>[+] <u>in "state 2"</u> [meaning: through slit 2]"

Mathematically, shows up as "superposition" or "linear combinations"

[+] Pay attention to "AND" here. This "AND" is at the heart of QM, and it describes situations that are unthinkable in classical mechanics. Also, "AND" is different from "OR".

Measurements play a special role

Role of measurement

- Electron is detected at one location (thus as a particle) on screen
- State of electron (right after measurement) becomes

  "it has a position $\vec{r}_{\text{measured}}$"

  [Key point: No longer $(\Psi_1 + \Psi_2)$ immediate after measurement]

* ∴ Measurement in QM changes the state (wavefunction) of a system.

Question: How does the wavefunction look like after position measurement?

"Collapse of Wavefunction" due to measurement

---

* This is a complete contrast with classical mechanics, where measurement is meant to find out a pre-existing value and measurement does not change the state of a system

This completes the discussion on how experiments using electrons reveal the **wave nature of particle**, and how the experimental facts had led us to

- The need of using a **wave description** for particles
- Introduce **wavefunction** as the quantity that describes a state of a system
- An interpretation of the wavefunction as a **probability amplitude** (wavefunction squared as probability density)
- Unusual meaning (c.f. classical physics) embedded in wavefunction
- Special role of measurement

This is a large part of Quantum Mechanics. Take this with you as we move on.

But something is missing?

What is the underlying wave theory? What is the wave equation in quantum mechanics? Only when we have the wave equation, we can calculate the wavefunction and then make use of the interpretation! That's the rest of the course!

Extension: More on 2-slit and multiple-slit experiments

$$\lambda = \frac{h}{p} \leftarrow \underbrace{\approx 10^{-34}\,J\cdot s}_{\text{tiny}}$$

$$E = \frac{p^2}{2m} \quad (\text{k.e., non-relativistic}) \Rightarrow \quad p = \sqrt{2mE}$$

$$\boxed{\lambda = \frac{h}{\sqrt{2mE}}}$$

- h is tiny
- $\sim \frac{1}{\sqrt{m}}$
- Macroscopic objects: atoms/molecules/"cat"
  λ is so tiny that wave nature can hardly be seen
- Early experiments use <u>electrons</u> (lightest particle)
- FoR many years, physicists have been trying to show that bigger objects still show the same behavioR in 2-slit expts

Later developments in experiments: Throwing bigger “particles” in 2-slit experiments

Wave–particle duality of C_{60} molecules

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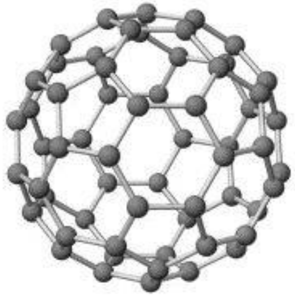
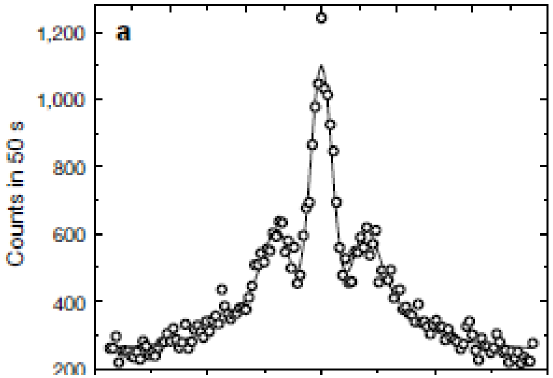
Institut für Experimentalphysik, Universität Wien, Boltzmanngasse 5, A-1090 Wien,

Nature 401 (1999) 680

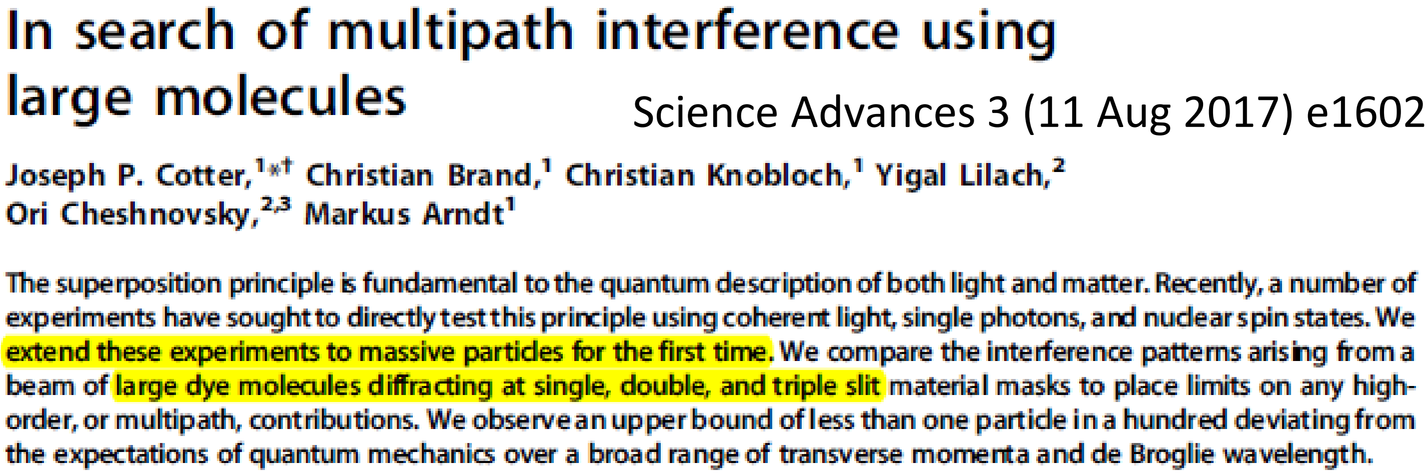


Throw this at double-slit apparatus



Position on detector

Still see 2-slit pattern



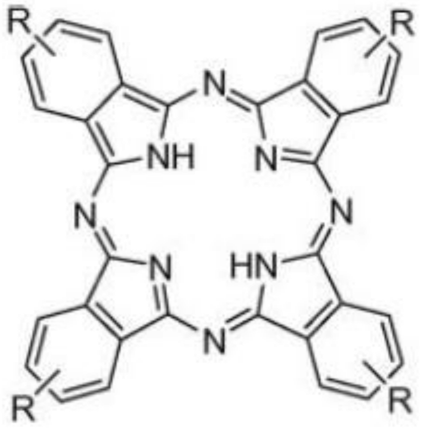
In search of multipath interference using large molecules

Science Advances 3 (11 Aug 2017) e1602478

Joseph P. Cotter,[1*†] Christian Brand,[1] Christian Knobloch,[1] Yigal Lilach,[2]
Ori Cheshnovsky,[2,3] Markus Arndt[1]

The superposition principle is fundamental to the quantum description of both light and matter. Recently, a number of experiments have sought to directly test this principle using coherent light, single photons, and nuclear spin states. We extend these experiments to massive particles for the first time. We compare the interference patterns arising from a beam of large dye molecules diffracting at single, double, and triple slit material masks to place limits on any high-order, or multipath, contributions. We observe an upper bound of less than one particle in a hundred deviating from the expectations of quantum mechanics over a broad range of transverse momenta and de Broglie wavelength.

Use phthalocyanine (PcH_2) molecules of mass 515 atomic mass unit (big object)



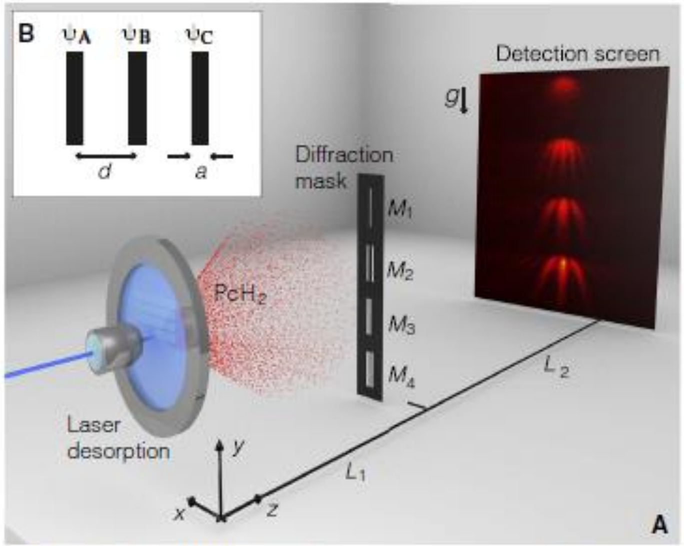
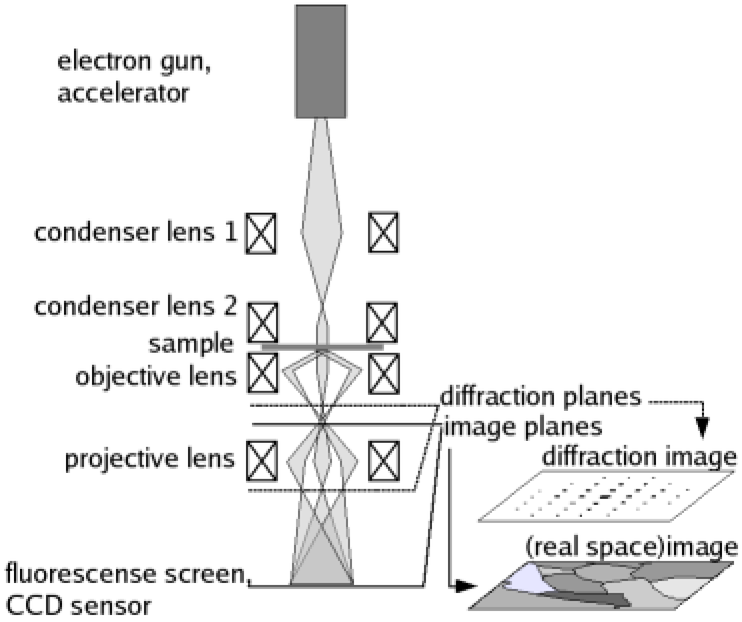


Fig. 1. Experimental setup. (**A**) Focused laser source produces a thermal beam of PcH_2 molecules, which diffracts at a vertical array of single, double, and triple slits, which are aligned to the local gravitation field g, before landing on a thin quartz detection screen. The deposited molecules are observed using high-resolution fluorescence imaging. (**B**) Schematic of the triple slit. The openings (black) have a transverse width $a = 80$ nm, and their centers are separated by a distance $d = 100$ nm.

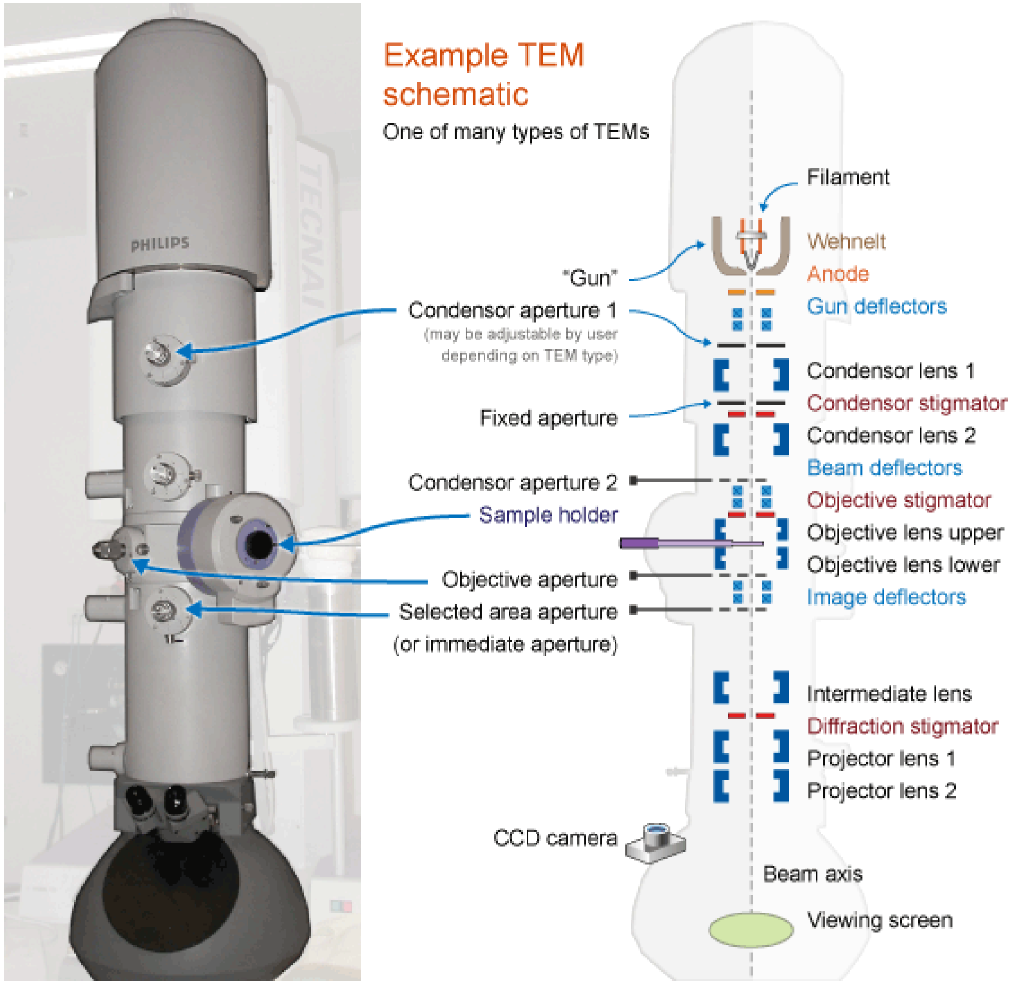
Results are all consistent with what we found in past 100 years!

Wave nature of particles is real and useful

Wave nature of electrons has led to the development of useful tools in research



An example is TEM (Transmission electron microscope) – can't do sample characterization without it!



Summary

- Unusual energies (nhf) in oscillators [Planck, Einstein (heat capacity)
- Particle nature of light [Einstein, Compton]
    - 2-slit exp't with dim source
        - What theory (wave theory) can do and cannot do
- Wave nature of particles [de Broglie, various experiments]
    - Need for wave description
    - $\Psi(\vec{r},t)$
    - Physical meaning attached to $|\Psi(\vec{r},t)|^2$
    - Unusual meaning of Ψ
    - Special role of measurements
    - $\lambda = \dfrac{h}{p} = \dfrac{2\pi\hbar}{p}$ [particle with definite momentum has definite wavelength]

Wave property → λ

particle property → p